

Matlab Tutorial for Basic Matrix Operations and Force Analysis

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My Office Hours

- * Location: JEC 2001
- * Monday: 4:00 – 5:00
- * Tuesday: 1:00 – 5:00
- * Friday: 1:00 – 4:00

Row Vector

- * Use space or comma to separate elements
- * E.g.

```
>> A=[1 2 3]
```

```
A =
```

```
    1    2    3
```

```
>> B=[4,5,6]
```

```
B =
```

```
    4    5    6
```

Column Vector

- * Use semicolon to separate elements
- * E.g.

```
>> C=[7;8;9]
```

```
C =
```

```
7  
8  
9
```

Matrix

- * Matrix can be viewed as a column vector of row vectors
- * Use semicolon to separate rows, and use space or comma to separate elements in a row
- * E.g.

```
>> D=[1 2 3;4,5,6]
```

```
D =
```

```
    1    2    3  
    4    5    6
```

Vector and Matrix

- * Row vector is a special case of matrix: only one row
- * Column vector is a special case of matrix: only one column
- * Scalar is also a special case of matrix: only one row and only one column

Element of Matrix

- * If A is a matrix, then $A(i,j)$ is the element in the i^{th} row and j^{th} column
- * E.g.

```
>> A=[1 2 3 4;5 6 7 8;1 3 5 7]
```

```
A =
```

```
     1     2     3     4
     5     6     7     8
     1     3     5     7
```

```
>> A(3,2)
```

```
ans =
```

```
     3
```

Submatrix

- * If A is a matrix, then $A(i:j,m:n)$ is the submatrix of A , which is the intersection of i^{th} row to j^{th} row of A and m^{th} column to n^{th} column of A (colon means “to”)
- * E.g.

```
>> A=[1 2 3 4;5 6 7 8;1 3 5 7]
A =
     1     2     3     4
     5     6     7     8
     1     3     5     7
>> A(2:3,1:2)
ans =
     5     6
     1     3
```


Basic Operations of Matrix

- * Addition: $A+B$
- * Subtraction: $A-B$
- * Scalar multiplication: $a*A$
- * Element by element multiplication: $A.*B$
- * Matrix multiplication: $A*B$
- * Power of matrix: A^n
- * Determinant of matrix: $\det(A)$

Example

- * Use Matlab to verify that $AB=BA$ is not necessarily true
- * Observe the difference between $A.*B$ and $A*B$
- * E.g. Use

```
A =  
    1    2  
    3    4  
  
B =  
    4    3  
    2    1
```

Example

```
>> A=[1 2;3 4]
```

```
A =
```

```
    1    2  
    3    4
```

```
>> B=[4 3;2 1]
```

```
B =
```

```
    4    3  
    2    1
```

```
>> A*B
```

```
ans =
```

```
    8    5  
   20   13
```

```
>> B*A
```

```
ans =
```

```
   13   20  
    5    8
```

```
>> A.*B
```

```
ans =
```

```
    4    6  
    6    4
```

Transpose

- * Transpose of A: A'
- * Transpose of a row vector is a column vector, vice versa

```
>> A=[1 2;3 4;5 6]           >> u=[1 2 3]
A =
     1     2
     3     4
     5     6
>> A'
ans =
     1     3     5
     2     4     6
>> u'
ans =
     1
     2
     3
```

Norm and Size

- * `norm()` computes the magnitude of a vector
- * `size()` gives the dimensionality of a matrix
- * E.g.

```
>> u=[1;1]           >> A=[1 2 3;4 5 6]       >> size(A)
u =
     1
     1
A =
     1     2     3
     4     5     6
ans =
     2     3

>> norm(u)           >> size(u)
ans =
    1.4142
ans =
     2     1
```

Dot (Inner) Product of Vectors

- * `dot(u,v)`
- * If both u and v are column vectors, we can also use matrix multiplication: $u' * v$

```
>> u=[1;2;3]
u =
     1
     2
     3
>> v=[3;2;1]
v =
     3
     2
     1
>> dot(u,v)
ans =
     10
>> u'*v
ans =
     10
```

Example: Angle and Projection

- * $u=(2,0,0)$, $v=(3,3,3)$
- * Use Matlab to compute
 1. The angle between u and v in degrees
 2. The projection of v on u

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

$$\text{proj}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|^2} \mathbf{u}$$

Example: Angle and Projection

```
>> u=[2;0;0]
```

```
u =
```

```
2  
0  
0
```

```
>> v=[3;3;3]
```

```
v =
```

```
3  
3  
3
```

```
>> angle=acosd(dot(u,v)/(norm(u)*norm(v)))
```

```
angle =
```

```
54.7356
```

```
>> proj_v_on_u=dot(u,v)/norm(u)^2*u
```

```
proj_v_on_u =
```

```
3  
0  
0
```


Cross Product and Moment

- * $\text{cross}(u,v)$ computes the cross product of u and v
- * E.g.
- * $F=(3,1,1)$ is a force on point B, the vector from B to point A is $(2,0,0)$, what is the moment of F about point A?

Cross Product and Moment

- * $F=(3,1,1)$ is a force on point B, the vector from B to point A is $(2,0,0)$, what is the moment of F about point A?

```
>> F=[3,1,1]

F =

     3     1     1

>> B_A=[2,0,0]

B_A =

     2     0     0

>> moment=cross(F,B_A)

moment =

     0     2    -2
```

Cross Product and Moment

* $P=(2,3,1)$, $Q=(4,0,3)$, $R=(6,1,0)$, what is $P \times (Q \times R)$?

```
>> P=[2 3 1]
P =
     2     3     1
>> Q=[4 0 3]
Q =
     4     0     3
>> R=[6 1 0]
R =
     6     1     0
>> cross(P,cross(Q,R))
ans =
    -6    -11    45
```

Linear Equations

- * $x_1 + 3x_2 = 9$
- * $2x_1 + x_2 = 8$

- * First method:
- * $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $b = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$
- * Equations: $Ax = b$
- * Solution: $x = A^{-1}b$

```
>> A=[1 3;2 1]
```

```
A =
```

```
    1    3  
    2    1
```

```
>> b=[9;8]
```

```
b =
```

```
    9  
    8
```

```
>> x=inv(A)*b
```

```
x =
```

```
    3  
    2
```

```
>> x=A^(-1)*b
```

```
x =
```

```
    3  
    2
```

Linear Equations

- * $x_1 + 3x_2 = 9$
- * $2x_1 + x_2 = 8$

- * Second method:
- * `rref()`: reduced row echelon form
- * $B = [1 \ 3 \ 9; 2 \ 1 \ 8]$, $x = [x_1; x_2]$
- * B is the augmented matrix

```
>> B=[1 3 9;2 1 8]
```

```
B =
```

```
    1    3    9
    2    1    8
```

```
>> B=rref(B)
```

```
B =
```

```
    1    0    3
    0    1    2
```

```
>> x=B(:,end)
```

```
x =
```

```
    3
    2
```

Exercise

* Solve:

$$* 2x_1 + x_2 - 4x_3 = 5$$

$$* 3x_1 + 8x_3 + 10x_4 = 7$$

$$* x_1 - 5x_2 + 9x_3 + 3x_4 = 8$$

$$* 5x_1 + 3x_3 + 6x_4 = -15$$

Key to Exercise

```
>> A=[2 1 -4 0;3 0 8 10;1 -5  
9 3;5 0 3 6]
```

```
A =
```

```
     2     1    -4     0  
     3     0     8    10  
     1    -5     9     3  
     5     0     3     6
```

```
>> b=[5;7;8;-15]
```

```
b =
```

```
     5  
     7  
     8  
    -15
```

```
>> x=inv(A)*b
```

```
x =
```

```
   -12.6680  
   -17.0810  
   -11.8543  
    13.9838
```